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On the Theory of Supersonic Inviscid Flow Separation in Gasdynamic Problems

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Abstract—A general schematic flow representation that explains the mechanism of inviscid gas separation in time-dependent and three-dimensional gas flows is presented. The scenario of gas flow separation from a body surface or a mixing layer is described as a vortex which induces in the flowfield a velocity opposing to that of the main flow, thus decelerating it. Within the framework of this scenario the analytical conditions of separation are obtained for conical and self-similar gas flows which coincide with the results of experimental and numerical simulations.

Keywords: shock waves, separation, Kelvin–Helmholtz instability, self-similar and conical flows.

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So far, the question about the nature of the forces inducing ideal gas separation and the process of formation of these forces has arisen amidst the specialists in gasdynamics. There are no problems in real gases: here, the viscous forces decelerate the gas flowing over a body surface and the elevated pressure "pushes off" the boundary layer from the surface. In the problems, where an ideal gas separates, the gas flow also decelerates but the deceleration mechanism is different: under certain initial or boundary conditions in the flow there arises a vortex which induces in the flowfield a velocity opposing to that of the oncoming (main) flow, thus decelerating it. The intensity of this vortex can depend both on the body corner shape [1, 2] and on the angle of yaw [3–5], that is, on the forces which push it from the surface. Depending on the vortex intensity, it floats up at one or another height relative to the body surface or the medium line with zero longitudinal velocity in the mixing layer. In the linear formulation this vortex represents a vortical singularity of the Ferri point type. It should be noted that the class of separated flows induced by inertia forces is considerably wider than that due to viscosity forces. Most frequently, these separations occur on collision of gas flows with different densities, when tangential or contact discontinuities are formed and roll into a vortex under the action of the appearing moments of forces. This can occur both on a body surface [1–5] and in an open space, as, for example, on the generation of tornados. Another example is furnished by the complicated pattern of the formation of vortices (microseparations) in a mixing layer subjected to a weak harmonic wave, when the arisen forces alternatively "push" the generated vortices downward and upward relative to the central mixing layer and lead to flow turbulization [6, 7]. In the case of an incompressible fluid the forces inducing inviscid separation are the source of the tangential discontinuity instability described in the classical Kelvin–Helmholtz problem [8]. In the natural conditions (in problems with account for viscous effects) separations produced by both viscous and inviscid effects can be simultaneously observable. In this case, the separations at the expense of the inertia forces are global. Thus, as shown in [4], provided that the preconditions for inviscid separation are created, it is realized above the boundary layer and the vortices produced by boundary layer separation.

In this study, we present the results concerning separation of a two-dimensional unsteady gas flow over a convex corner. The results of analytical and numerical investigations are presented and compared with the

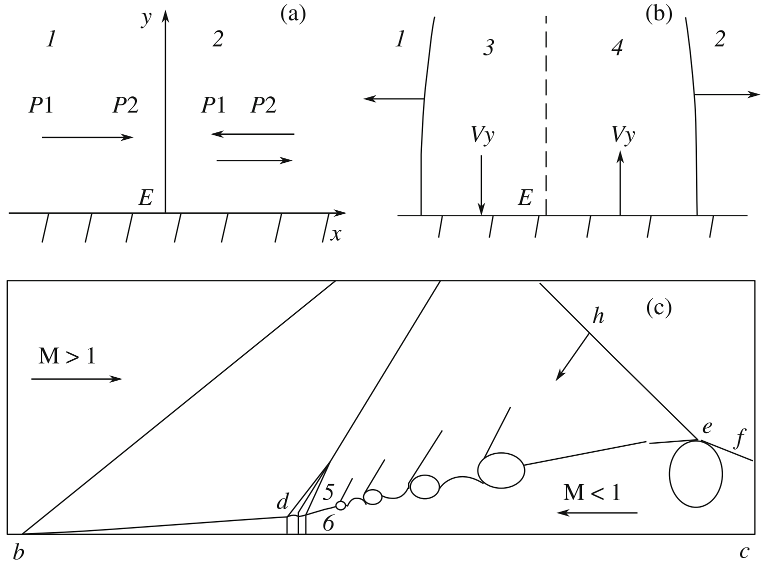

Fig. 1. Schematic representation of the formation of the forces inducing inviscid gas flow separation; (a) on the interaction between counter-streaming flows; (b) formation of a tangential discontinuity on a body surface; and (c) in the mixing layer.

experiment. The separation pattern proposed in the study is shown to give an exact solution of problem [2].

The second problem analyzed pertains to steady three-dimensional flow past a V-wing solved in [3–5], where the existence of separated flow was experimentally and numerically shown for large angles of attack and yaw. For this problem the scheme proposed makes it possible to analytically derive the separation conditions obtained experimentally and numerically in the above-mentioned studies.

The results obtained within the framework of the ideal gas model extend our theoretical knowledge about the mechanisms of separated flow development. At the same time, they are of practical interest in explaining the results obtained in numerically solving the problem of flow past bodies.

1. FORMULATION OF THE PROBLEM

The mechanism of the generation of forces inducing vortex formation in an inviscid gas flow is proposed and the known solutions are analyzed on its basis (Fig. 1). It is shown that in the vicinity of point E on the body surface flows with different parameters are generated in regions *1* and *2* (Fig. 1a), when initial or boundary conditions of the problem are changed. At the next moment, in accordance with the conservation laws [9], regions *3* and *4* separated by a tangential discontinuity are formed on collision of the flows (Fig. 1b). Here, it is not important in which region point E lies but of fundamental importance is the behavior of the derivative V_y normal to the surface in regions *3* and *4*. In most cases its sign does not reverse and then separation is absent. When the sign of the derivative changes, the flow detaches from the surface in region *4* (as in the figure) and attaches to it in region *3*. Therefore, at the expense of the pressure difference in the vicinity of point E a force rolling the tangential discontinuity is generated. As a result, a vortex is formed and floats up above the surface. A more complicated pattern of the separated flow formation takes place in the case of a mixing layer.

In Fig. 1c the pattern of vortex formation is presented for the problem of the shock wave *he* incidence on a flat plate *bc* in a supersonic gas flow [7]. The problem is time-dependent and the fluctuations arising on the shock wave passage through the vortices arrive in the form of compression waves at the boundary layer separation point *d* through its subsonic part. On the interaction between the boundary layer and the mixing layer (the upper boundary of the separated boundary layer) vortices are generated. An analysis of the flow pattern shows that the vortices oscillate near the central mixing layer and, correspondingly, the quantity V_y changes the sign at a point within the mixing layer.

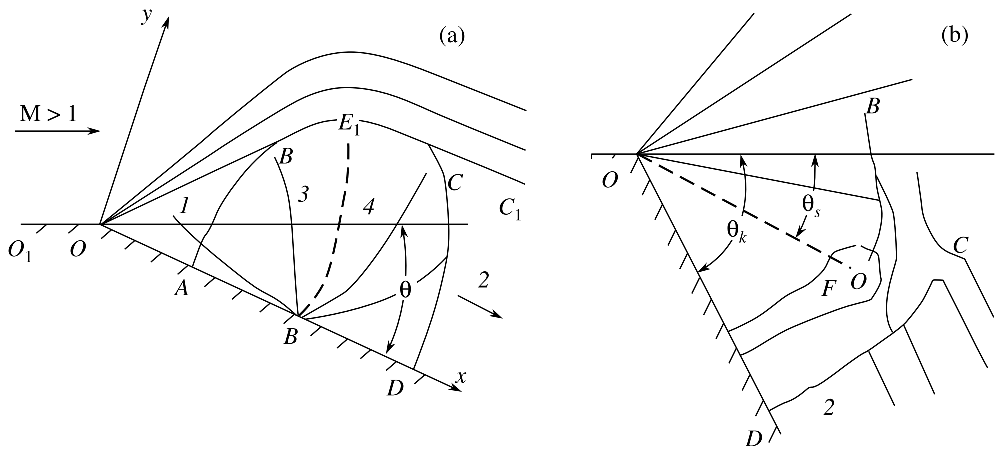

Fig. 2. Separated unsteady gas flow around a convex corner; (a) separationless flow and (b) separated flow.

2. INVISCID GAS SEPARATION IN THE TIME-DEPENDENT PROBLEM OF FLOW PAST A CONVEX CORNER

We will consider the problem of sudden gas flow past a convex corner (Fig. 2), when the gas starts suddenly flow along the side O_1O at a supersonic velocity Q. The main problem parameters are the pressure P, the density ρ, the entropy S, and the gas flow velocity Q, as well as the Mach number $\mathrm{M} = Q/\sqrt{P\gamma/\rho}$ and the adiabatic exponent γ. The quantities P and ρ are normalized by their freestream values; in any flow region the parameters are taken with the subscript corresponding to its number. The problem is self-similar; for this reason, in what follows we will consider the flow patterns at the moment $t = 1$ for different values of the angle θ. In the case of small θ the disturbed gas region ABE_1CD (Fig. 2a) is bounded by the limiting characteristics AB and CD and the isobaric curve BC. Inside the region there is a weak contact discontinuity EE_1. For the given M and θ the gas parameters in regions *1* and *2* can be determined from the well-known formulas [8]. The tangential velocity along the surface in region *2* (relative to the vertex O) is $Q_2 = Q\cos\theta$, while the entropy values in regions *1* and *2* are equal to its freestream value.

With increase in θ the flow parameters in regions *1* and *2* start to considerably differ: the velocity Q_1 increases, whereas Q_2 diminishes. The region $ABCD$ is bounded by strong discontinuities AB and DC. Inside the region a tangential discontinuity EE_1 of finite intensity is formed. The flow one-dimensionality in the wall region makes it possible to determine the flow parameters in regions *3* and *4* from the formulas of the arbitrary one-dimensional discontinuity breakdown [9]. The point E is singular, of the Ferri point type [10]; here, at small values of θ isentropic lines converge, including the isentrope EE_1 along which the total velocity suffers a discontinuity (Fig. 2a). The presence of a finite velocity discontinuity leads to the formation of vortex F in the vicinity of point E. With increase in θ to 55° the vortex F floats up above the surface (Fig. 2b). The gas rotating clockwise around the vortex F in the wall region AE partially decelerates the gas flowing from point A to point E. With increase in θ the intensity and velocity of the shock wave AB increase, so that its velocity relative to the vertex becomes equal to the gas flow velocity in region *1*: $D_{13} = Q_1$. Precisely this condition determines the value of the angle θ_k at which the oncoming gas stream separates from the vertex. The angle θ_s is determined from the condition $\mathrm{M}_3 = 1$, which in the self-similar variables describes the floating-up of the vortical Ferri singularity and is in agreement with the experimental data [11] and the results of the numerical modeling [1, 2]. Thus, there are two conditions that allow one to determine the angles θ_k and θ_s, namely $D_{13} = Q_1$ and $\mathrm{M}_3 = 1$. They are necessary for gas flow separation from the corner vertex: firstly, the disturbances from the base region at preseparation angles must reach the corner vertex and, secondly, the flow in the separation zone must be subsonic.

We will consider how the inviscid separation pattern proposed is applicable in our problem. We will determine the normal derivative on the body surface in the vicinity of point E and analyze its behavior on the basis of the time-dependent Euler equations.

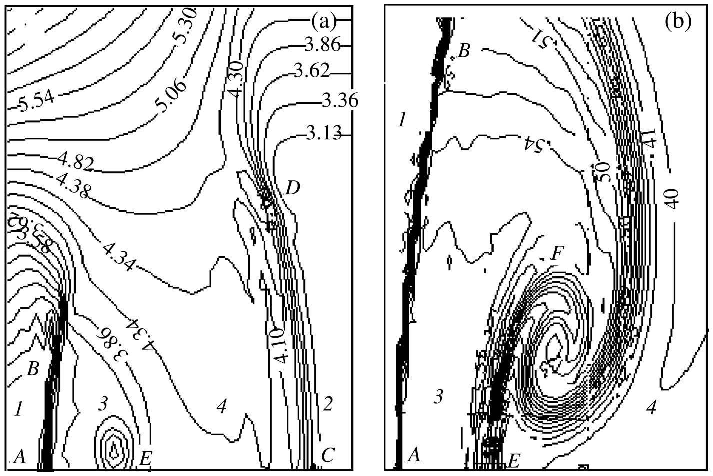

Fig. 3. Vortex formation in the time-dependent problem; small (a) and large (b) values of the vertex angle of the convex corner.

In the self-similar variables $\xi = x/t$ and $\eta = y/t$ the equations of continuity, momentum, and entropy conservation take the form [8]:

$$
\begin{aligned}
&(\rho U)_\xi + (\rho V)_\eta + 2\rho = 0,\\
&UU_\xi + VU_\eta + U + P_\xi/\rho = 0,\\
&UV_\eta + VV_\eta + V + P_\eta/\rho = 0,\\
&US_\xi + VS_\eta = 0,
\end{aligned}
\tag{2.1}
$$

where U and V are the velocity components in the movable $(\xi,\ \eta)$ coordinate system, which are related with the velocity components in the $(x,\ y)$ coordinate system by the equations $U = u - \xi$ and $V = v - \eta$.

We will consider the flow in the vicinity of point E (Fig. 2a). On the body surface the normal velocity component $V = 0$ and the equations are simplified

$$
\begin{aligned}
&\rho_\xi U + \rho(U_\xi + V_\eta + 2) = 0,\\
&U(1 + U_\xi) + P_\xi/\rho = 0,\\
&UV_\xi + P_\eta/\rho = 0,\\
&P_\xi = a^2\rho_\xi.
\end{aligned}
\tag{2.2}
$$

Substituting the quantity U_ξ from the second equation (2.2) into the continuity equation we obtain

$$
V_\eta = -1 - P_\xi(\mathrm{M}_3^2 - 1)/\rho U,
\tag{2.3}
$$

where the quantity $\mathrm{M}_3 = U/a$ is the Mach number on the OD surface in region *3*.

As can be seen from Eq. (2.3), the quantity $V = -\eta$ at $\mathrm{M}_3 = 1$. Then from the definition $V = v - \eta$ it follows that in the fixed coordinate system $(x,\ y)$ the normal velocity component $v = 0$. At $\mathrm{M}_3 \neq 1$ the behavior of the quantity P_ξ must be taken into account.

An analysis of the pressure behavior in the vicinity of point E shows that for θ similar in value with θ_s the quantity $P_\xi > 0$. Then from Eq. (2.3) it follows that at $\mathrm{M}_3 > 1$ $v_y < 0$, that is, the singular point squeezes up against the surface OD. Contrariwise, at $\mathrm{M}_3 < 1$ we have $v_y > 0$ and the singular point floats up above the surface. This can be seen in Fig. 3 in which the density fields are presented for the cases in which the angle θ is greater than θ_s by a small value (Fig. 3a) and the angle $\theta \approx \theta_k$ (Fig. 3b). In our case, in region *4* the quantity $\mathrm{M}_4 > 1$, while in region *3* $\mathrm{M}_3 < 1$, so that, in accordance with the separation scheme, a compression wave occurs in region *4* and an expansion wave is generated in region *3*. This force rolls up the tangential discontinuity with the formation of a vortex which floats up above the surface and induces a velocity field rotating clockwise, counter the main stream.

Thus, in this time-dependent problem the change of the sign of the normal derivative of the velocity to the body surface leads to flow separation from the surface, while the condition $M_3 = 1$ ($v_y = 0$) determines the flow separation angle θ_s which is the exact solution of the problem.

3. SEPARATION OF THE THREE-DIMENSIONAL GAS FLOW AROUND A V-WING

We will consider a V-wing with straight edges AB and BC and the central chord $B'E$ (see Fig. 4, in which a section located at a certain distance from the leading edges is presented). The vee angle ABC of the wing is 90°.

Let a supersonic flow at a velocity U_0 be incident on the wing set at the angles of attack α and yaw β. The velocity can be expanded into the components $B'F$, or the velocity directed along the central chord, and DF, or the transverse velocity perpendicular to BE. Owing to the presence of the transverse velocity the flow around the wing is nonsymmetric. In this case, the intensities of the shocks Ab and Cc are different. The gas flow along the cantilever AB moves toward the chord BE at a greater velocity than along the cantilever BC. Thus, with increase in the yaw angle (with increase in the intensities of the contact discontinuities bS' and cS') the point S' progressively removes from point B with which it coincides in the symmetric wing flow. At certain angles of attack and yaw point S' separates from the surface [3–5]. The characteristic regions formed in this problem on the interaction between gasdynamic discontinuities are marked by numbers *1* to *5* in Fig. 4. We consider the regimes of flow around wings with supersonic edges.

In Fig. 5 for the initial parameters of the problems ($M = 3$, $\alpha = -35°$, and $\beta = 13°$) the results for the flow around the V-wing are presented in the case in which on the windward wing side there occurs separation in the form of the floating-up of the "vortical" Ferri singularity (point S'). The transverse distribution of the gasdynamic functions over the cantilever surfaces is presented in Fig. 5a, where the vertical line *6* presents the separation boundary. Here, the general flow pattern (Fig. 5b, the entropy field) is presented, together with the fragments of the isentrope fields in the flow region *4* (Fig. 5c). Clearly that in the separation region a local pressure P maximum is realized (curve *1*), while the density ρ (*2*) and the entropy S (*3*) vary jumpwise. In this case, the transverse velocity $u(4) = 0$. The broken lines *5* on the edges of the left and right cantilever mark the values of the pressure and the density obtained from the exact relations on oblique shocks.

In accordance with the separation scheme proposed, we will determine the conditions under which the separation flow is realized in this problem.

We will write the equation of gas motion in the spherical coordinate system R, θ, φ with the velocity components u, v, and w directed along these axes [10]; in Fig. 1b it is shown how this coordinate system is fitted to the wing surface. For the conical flow, with account for the fact that $d/dR = 0$ along a ray, we obtain

$$\begin{aligned}
&(\rho v \sin\theta)_\theta + (\rho w)_\varphi + 2\rho u \sin\theta = 0,\\
&v u_\theta + w \csc\theta u_\varphi - v^2 - w^2 = 0,\\
&v v_\theta + w \csc\theta v_\varphi + uv - w^2 \cot\theta = -P/\rho,\\
&(v w_\theta + w \csc\theta w_\varphi)\sin\theta + w(u\sin\theta + v\cos\theta) = -P_\varphi/\rho,\\
&v S_\theta + w \csc\theta S_\varphi = 0.
\end{aligned} \tag{3.1}$$

We will write Eqs. (3.1) on the wing surface, where $w = 0$. As a result, we obtain

$$\begin{aligned}
&(\rho v \sin\theta)_\theta + \rho w_\varphi + 2\rho u \sin\theta = 0,\\
&v u_\theta - v^2 = 0,\\
&v v_\theta + uv = -P_\theta/\rho,\\
&v w_\theta \sin\theta = -P_\varphi/\rho,\\
&v S_\theta = 0.
\end{aligned} \tag{3.2}$$

We will express the derivative w_φ normal to the wing surface in terms of the flow parameters. Depending

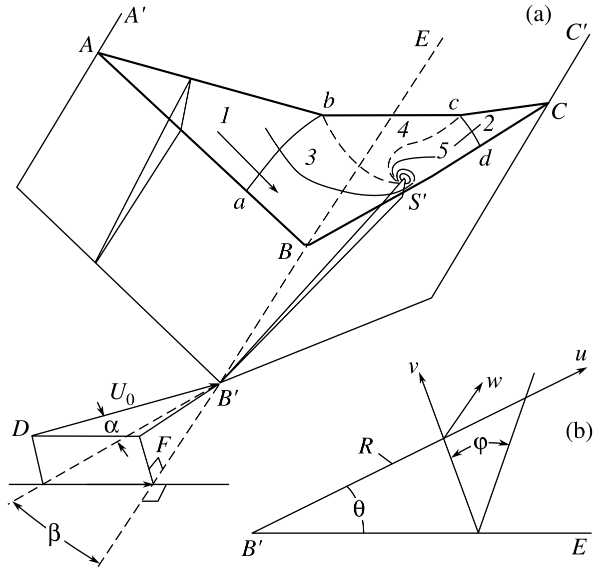

Fig. 4. General pattern of flow around a V-shaped wing; (a) gasdynamic discontinuities (*Ab*, *bc*, *cC*, *ab*, and *cd* are shock waves and bS' and cS' are tangential discontinuities) and characteristic regions (*1–5*) and (b) spherical coordinate system.

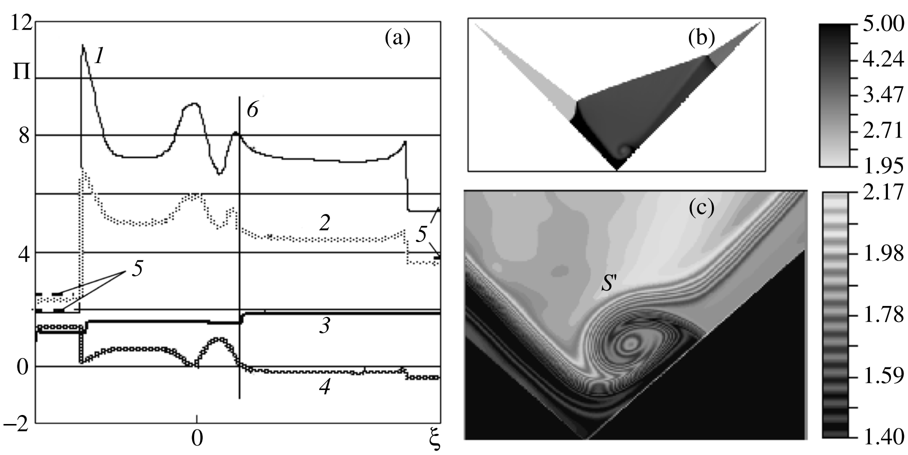

Fig. 5. Results of the calculations of separated flow around a wing at $M = 3$, $\alpha = -35°$, and $\beta = 13°$; (a) flow parameter distributions over the cantilever surfaces in the transverse direction; $\Pi = P$ (curve *1*), ρ (*2*), S (*3*), and u (*4*); (*5*) is the comparison of the pressure and the density with their exact values, and (*6*) is the separation boundary; (b) entropy field; and (c) fragments of the entropy field.

on the sign of this derivative, the gas either squeezes up against or detaches from the surface. Since the vortex singularity moves with the gas velocity, this behavior is also pertinent to it.

From the first equation (3.2) we obtain

$$w_\varphi = -\sin\theta(\rho_\theta v/\rho + v_\theta + 2u) - v\cos\theta. \quad (3.3)$$

Using the third equation and the condition $P_\theta = a^2\rho_\theta$, where a is the speed of sound, from the fifth equation (3.2) we will determine the quantity ρ_θ and substitute it into Eq. (3.3). Finally, since in the conical flow the relation $u\sin\theta + v\cos\theta = 0$ is fulfilled along a ray [10], we determine w_φ in the form:

$$w_\varphi = -\sin\theta(v_\theta + u)(1 - v^2/a^2). \quad (3.4)$$

From Eq. (3.4) we obtain that $w_\varphi = 0$, when either $1 - v^2/a^2 = 0$ or $v_\theta + u = 0$. The calculations showed that the former expression does not turn to zero. From the equality of the latter expression to zero it follows that in the third equation of system (3.2) the quantity $P_\theta = 0$. Therefore, in this case a local extremum is realized on a ray lying on the wing surface. In fact, the data of [3, 4] and the results of the numerical modeling in this study show that here a pressure maximum is realized.

An analysis of the separation pattern in Fig. 5 and the data of [1–3] indicates that the mechanism of inviscid gas separation from a surface is the same in the cases of three-dimensional conical and two-dimensional time-dependent (self-similar) gas flows. In both cases, upon the collision of two gas flows there appears a velocity component normal to the surface, which separates the vortical Ferri singularity from the surface. This vortex, whose intensity depends on the angles α and β and the Mach number M, induces on the windward side of a cantilever a velocity directed oppositely to that of the gas flowing from the cantilever sides.

Summary. The mechanism of the formation of the forces leading to separation of supersonic inviscid gas flows in gasdynamic problems is schematically described. In accordance with this scheme, the gas flow separation conditions are obtained for the problems of time-dependent and three-dimensional flows past bodies. They correspond to the results of numerical and experimental investigation.

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