

*To the memory of O.M. Belotserkovskii*

# Numerical Simulation of Spinning Detonation in Circular Section Channels

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**Abstract**—Numerical simulation of three-dimensional structures of gas detonation in circular section channels that emerge due to the instability when the one-dimensional flow is initiated by energy supply at the closed end of the channel is performed. It is found that in channels with a large diameter, an irregular three-dimensional cellular detonation structure is formed. Furthermore, it is found that in channels with a small diameter circular section, the initially plane detonation wave is spontaneously transformed into a spinning detonation wave, while passing through four phases. A critical value of the channel diameter that divides the regimes with the three-dimensional cellular detonation and spinning detonation is determined. The stability of the spinning detonation wave under perturbations occurring when the wave passes into a channel with a greater (a smaller) diameter is investigated. It is found that the spin is preserved if the diameter of the next channel (into which the wave passes) is smaller (respectively, greater) than a certain critical value. The computations were performed on the Lomonosov supercomputer using from 0.1 to 10 billions of computational cells. All the computations of the cellular and spinning detonation were performed in the whole long three-dimensional channel (up to 1 m long) rather than only in its part containing the detonation wave; this made it possible to adequately simulate and investigate the features of the transformation of the detonation structure in the process of its propagation.

**Keywords:** spinning detonation, cellular detonation, three-dimensional channel, numerical simulation, program package, supercomputer, numerical solution, Euler system of equations.

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## 1. INTRODUCTION

The study of detonation waves in gases is mainly stimulated by the desire to use them for practical purposes in impulse facilities and special energy systems designed for rockets and other flying vehicles. Large magnitudes of the gas dynamics parameters and the complex flow pattern behind the detonation wave front significantly complicate both the experimental and theoretical study of detonation. The main source of information about detonation waves is the experimental study. Among the experimenters, a special place is held by Soloukhin (see [1–6]). His works became handbooks for generations of researchers. A significant contribution to the study of detonation was made by his colleagues, disciples, and students. As experimental data were accumulated, theoretical detonation models were improved as well. For example, the two-stage model, which takes into account the ignition delay and a finite time of the subsequent heat emission, makes it possible to describe the nonstationary nonlinear wave structure of detonation (see [7]). In the framework of the infinitely thin detonation wave, the asymptotic nature of the transition of the plane wave to the Chapman-Jouguet regime was established; in the framework of the cylindrical and spherical model, it was found that this transition occurs at a finite distance from the point of the detonation wave formation (see [8, 9]). Within the two-stage model, the initial phase of the flow initiated by a point explosion was analytically investigated, and the effect of the detonation wave splitting was discovered (see [10]). Numerically, using the model and real-life kinetics, the mechanism of the formation and propagation of the self-sustaining detonation wave as a result of explosion was established; in particular, it was shown that such a wave is always nonstationary, and the parameters on its leading front vary periodically under the influence of the shock waves that occur in the induction zone ahead of the accelerating flame

front (see [11–15]). The self-oscillation process develops only in the case when the explosion energy magnitude exceeds a critical value. Otherwise, the detonation wave decays and disintegrates into a shock wave and a slow combustion wave. The critical values of energy in the cases when the detonation is initiated by a piston, electric discharge, exploding wire, or TNT charge were found, its dependence on the parameters of the combustible mixture and time–space characteristics of the energy sources was determined, and an explanation of the anomalous experimental dependence of the critical energy on the duration of the electric discharge was given (see [15–20]). The problem of reducing the critical initiation energy was considered, and novel mechanisms of excitation of detonation were investigated. In particular, it was discovered that the detonation can be initiated without supplying energy from outside (see [21–27]). In the framework of investigations of the non-one-dimensional detonation structure (initially using the two-stage kinetics), the development of the plane wave disturbance resulting in the formation of a cellular detonation structure was considered, the existence of the minimum and maximum size of cells was discovered, and the determining role of transverse waves in the initiation and propagation of detonation (in particular, when the wave passes into a divergent channel) was revealed (see [28]). The two-dimensional spin model, which is used by many researchers, was formulated, and the structure of the double-head spinning detonation wave was calculated (see [14]). The wave processes occurring in the detonation of hydrogen–air mixture in plane channels of complex shape with account for real kinetics were studied (see [29–33]). The initiation of detonation in the supersonic stream of hydrogen–air mixture by an electric charge with spatially homogeneous and inhomogeneous energy release was investigated (see [34, 35]).

In the recent decade, special attention has been paid to the problems of initiation and stabilization of detonation in the bounded combustion chamber of energy generating facilities that use high-rate burn of fuel. In this respect, a special role is acquired by mathematical modeling methods, which due to the rapid development of high-performance computers open almost unlimited possibilities for the study of various phenomena in science and engineering with account for complex high-rate physical and chemical processes. Note that a close collaboration of theoreticians and experimenters is needed for this purpose. Recent results on the initiation of detonation show that numerical experiments make it possible to find new flow patterns that guarantee the formation of self-sustaining detonation burn that make use of the mechanical energy of the combustible mixture. Important results concerning the cellular detonation structure that make a significant contribution to the solution of fundamental problems of detonation have been obtained.

The use of high-performance computers made it possible to study multi-dimensional flows related to the detonation due to the energy of the combustible mixture motion and its interaction with moving boundaries (see [36–46]). New regimes of the propagation of chemical reaction waves, including the galloping layered detonation, were discovered.

We especially note that an indispensable attribute of gas detonation is the presence of nonstationary compression shocks behind the leading shock front. The self-sustaining detonation wave can propagate only due to the interaction between those and their interaction the leading compression shock. When the detonation is simulated in the one-dimensional approximation with account for the finite duration of chemical reactions, longitudinal compression shocks that periodically form ahead of the self-accelerating heat release front are observed; in experiments and in computations using two- and three-dimensional approximation, these are transverse waves behind the leading shock.

## 2. THE MATHEMATICAL MODEL AND NUMERICAL METHOD

To describe gas-dynamic three-dimensional nonstationary flows, we use the Euler system of equations for the perfect multicomponent reacting mixture in a fixed Cartesian coordinate system, which in conservative form is

$$\begin{aligned} \frac{\partial \rho_i}{\partial t} + \frac{\partial(\rho_i u)}{\partial x} + \frac{\partial(\rho_i v)}{\partial y} + \frac{\partial(\rho_i w)}{\partial z} &= \omega_i, \\ \frac{\partial(\rho u)}{\partial t} + \frac{\partial(p + \rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z} &= 0, \\ \frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(p + \rho v^2)}{\partial y} + \frac{\partial(\rho vw)}{\partial z} &= 0, \end{aligned}$$

$$\begin{aligned} \frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho u w)}{\partial x} + \frac{\partial(\rho v w)}{\partial y} + \frac{\partial(p + \rho w^2)}{\partial z} &= 0, \\ \frac{\partial(H - p)}{\partial t} + \frac{\partial(Hu)}{\partial x} + \frac{\partial(Hv)}{\partial y} + \frac{\partial(Hw)}{\partial z} &= 0, \\ H = \sum_{i=1}^N \rho_i h_i + \rho \frac{u^2 + v^2 + w^2}{2}, \quad \rho &= \sum_{i=1}^N \rho_i. \end{aligned}$$

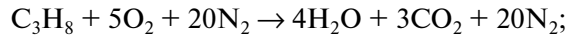
Here  $p$  and  $\rho$  are the pressure and density of the mixture;  $u$ ,  $v$ , and  $w$  are the velocity components along the axes  $x$ ,  $y$ , and  $z$ , respectively;  $N$  is the number of the mixture components,  $\rho_i$  and  $h_i$  are the density and enthalpy of the  $i$ th component;  $\omega_i$  is the rate of change of  $\rho_i$  under chemical reactions; and  $H$  is the total enthalpy.

The equations of state of the mixture have the form

$$p = \sum_{i=1}^N (\rho_i / \mu_i) R_0 T, \quad h_i = c_{0i} + c_{pi} T, \quad i = 1, \dots, N,$$

where  $T$  is the mixture temperature,  $\mu_i$  are the molar masses of the components,  $R_0$  is the gas constant, and  $c_{0i}$ ,  $c_{pi}$  are constant coefficients obtained by approximating the tabular values (see [47]).

To describe the chemical reactions in a combustible hydrocarbon–air mixture, one-stage kinetics (see [48]) with one irreversible reaction is used. We study the flows of the propane–air mixture in which the reaction proceeds according to the stoichiometric equation



here  $N = 5$ , and the reaction rate determines all  $\omega_i$  by the formulas

$$\begin{aligned} \frac{\omega_{\text{C}_3\text{H}_8}}{\mu_{\text{C}_3\text{H}_8}} = \frac{\omega_{\text{O}_2}}{5\mu_{\text{O}_2}} = -\frac{\omega_{\text{H}_2\text{O}}}{4\mu_{\text{H}_2\text{O}}} = -\frac{\omega_{\text{CO}_2}}{3\mu_{\text{CO}_2}} &= AT^\beta e^{-\frac{E}{R_0 T}} \left( \frac{\rho_{\text{C}_3\text{H}_8}}{\mu_{\text{C}_3\text{H}_8}} \right)^a \left( \frac{\rho_{\text{O}_2}}{\mu_{\text{O}_2}} \right)^b, \\ \omega_{\text{N}_2} &= 0, \end{aligned}$$

where the indices  $i$  are replaced with the symbols of the mixture components, and  $A$ ,  $E$ ,  $a$ ,  $b$ , and  $\beta$  are constants.

In the problems considered below, the air is assumed to be a mixture of oxygen and nitrogen in the molar proportion  $v_{\text{O}_2} : v_{\text{N}_2} = 1 : 4$ , and the propane–air mixture is given by the proportion  $v_{\text{C}_3\text{H}_8} : v_{\text{O}_2} : v_{\text{N}_2} = 1 : 5 : 20$ .

On the walls, the impermeability condition is set.

The investigation is performed using the modified Godunov method (see [49]) of the first-order with respect to space and time. It is implemented in a program novel package designed for solving a wide class of one- two-, and three-dimensional problems of nonstationary dynamics for gaseous combustible mixtures. To calculate the flows of reacting gas mixtures, high spatial resolution under which each reaction zone includes a large number of cells is required. For this reason, high-performance multiprocessor computers are needed for numerical simulations. The program package developed and used in the present study has a graphical interface with visualization functions (Fig. 1). The computational algorithm is parallelized using MPI. It makes it possible to solve problems using grids with billions of cells. In this paper, we present the results obtained on the Lomonosov supercomputer using grids with 0.1–10 billions of computational cells.

### 3. STATEMENT OF THE PROBLEM FOR THE THREE-DIMENSIONAL DETONATION STRUCTURE IN CIRCULAR SECTION CHANNELS

The developed program package was used for the detailed simulation of the detonation structure in three-dimensional circular section channels. It was assumed that in such channels, due to the smooth lateral surface, there are benign conditions for the formation of the spinning detonation regime under which the transverse wave propagates on a spiral simultaneously along the main front and along the channel surface.

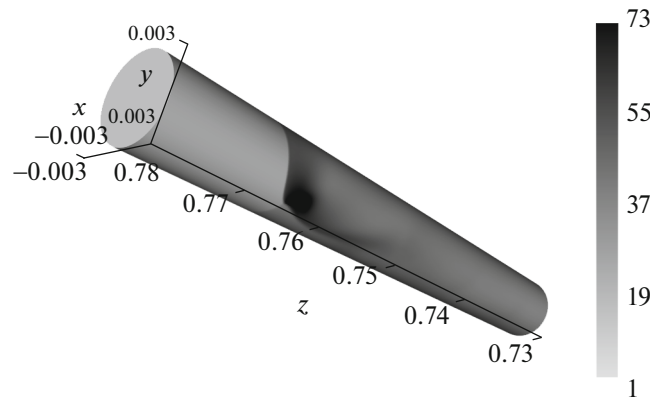


Fig. 1. Examples of data visualization for the numerically simulated spinning detonation.

Channels 1 m long of various diameters  $D$  varying in a wide range were considered. In all the cases, a special computational grid not condensing in the vicinity of the symmetry axis was used. The linear size of the cells did not exceed 0.1 mm. All the computations were performed for the entire channel rather than for its part containing the detonation wave, which made it possible to adequately simulate and investigate the features of the transformation of the detonation structure in the process of its propagation.

Detonation in the fixed stoichiometric propane–air mixture at the pressure of 1 atm and temperature of 20°C was investigated. It was assumed that the detonation is initiated by the instantaneous supply of energy in the three-centimeter zone near the closed end of the channel. Due to the problem statement, the flow is one-dimensional in the initial phase, and its parameters depend only on the longitudinal coordinate  $z$ . However, the instability of combustion behind the leading detonation wave front causes the increase of disturbances whose source is the roundoff errors at the machine precision level. As a result, the one-dimensional detonation wave gradually becomes three-dimensional. In the computations, the three-dimensional numerical trace pattern was composed using local pressure maximums. The values of the pressure maximums on the lateral channel surface that together form two-dimensional numerical trace patterns similar to the trace patterns of triple points registered in experiments, were also a subject of the study and were visualized.

In channels in which the section is sufficiently large for the propagation of a large number of transverse waves, spontaneous formation of an irregular three-dimensional cellular detonation structure with transverse waves propagating in the cross-section plane and interacting with each other and with the lateral channel surface was observed. The traces on the channel surface are distinct from diamond-shaped, and the trace pattern is chaotic even though it resembles the two-dimensional periodic structure observed in plane channels. The computations show that the irregularity of the three-dimensional cellular detonation is directly related to the existence of the additional (compared with the two-dimensional case) spatial degree of freedom. At the same time, the decrease in the transverse channel size results in the decrease of the number of transverse waves up to their complete suppression. From the general considerations, it is clear that when the channel width is less than a certain critical value, the detonation will always propagate in the mode close to the one-dimensional one. In addition, it is reasonable to expect that, in the presence of only one transverse wave, the flow pattern will not be irregular any more. The experimental results suggest that the detonation propagates in the spinning mode in this case.

The purpose of this paper is to numerically simulate the process of formation of spinning detonation caused by the instability of the one-dimensional flow, determination of the range of the channel diameter values in which the spinning detonation regime is realized, and to investigate the stability of the detonation, in particular, its transformation when the size of the channel's cross section varies. To investigate the stability of the spinning detonation, the following dependence of the section diameter  $D$  on the longitudinal coordinate  $z$  was used:  $D = D_1$  for  $0 < z < L$ ,  $D = D_1 + (z - L)(D_2 - D_1)$  for  $L < z < |D_2 - D_1|$ , and  $D = D_2$  for  $z > L$ . This corresponds to the channels consisting of two cylindrical sections connected by a conical section with the half-angle of 45°. For  $D_2 > D_1$ , we have a diverging channel and for  $D_2 < D_1$  a converging channel.

#### 4. NUMERICAL RESULTS FOR THE SPINNING DETONATION IN CYLINDRICAL CIRCULAR SECTION CHANNELS

Due to the statement of the problem, the mixture between the wall  $z = 0$  and the plane  $z = 3$  cm is ignited and burns down almost instantly. As a result of the arbitrary discontinuity breakdown, a one-dimensional plane detonation wave propagates through the mixture along the axis  $z$  in the positive direction. It was earlier shown by the authors of this paper that, in the case of a sufficiently large channel diameter, the motion of the one-dimensional detonation wave is accompanied, due to its instability, by the increase in small perturbations, which always occur in numerical computations. As a result, the one-dimensional detonation acquires an irregular three-dimensional cellular structure with time. In this paper, computations for various values of the channel diameter  $D$  are described, and the ranges of this parameter corresponding to different flow regimes are determined. The numerical results show that the detonation remains practically one-dimensional for  $D < 2$  mm. For  $2 \text{ mm} < D < 4.5$  mm, weak transverse waves occur in due time, but their intensity is insufficient for forming a clear flow pattern and no spinning detonation is formed even at the distance of 1 m from the end wall. The computations show that, in the case  $D > 9$  mm, the detonation turns into the irregular three-dimensional cellular structure, which is observed from the time of its formation to the time when it moves off to the distance of 1 m from the end wall. For  $4.5 \text{ mm} < D < 9$  mm, the one-dimensional detonation gradually transforms into the spinning detonation.

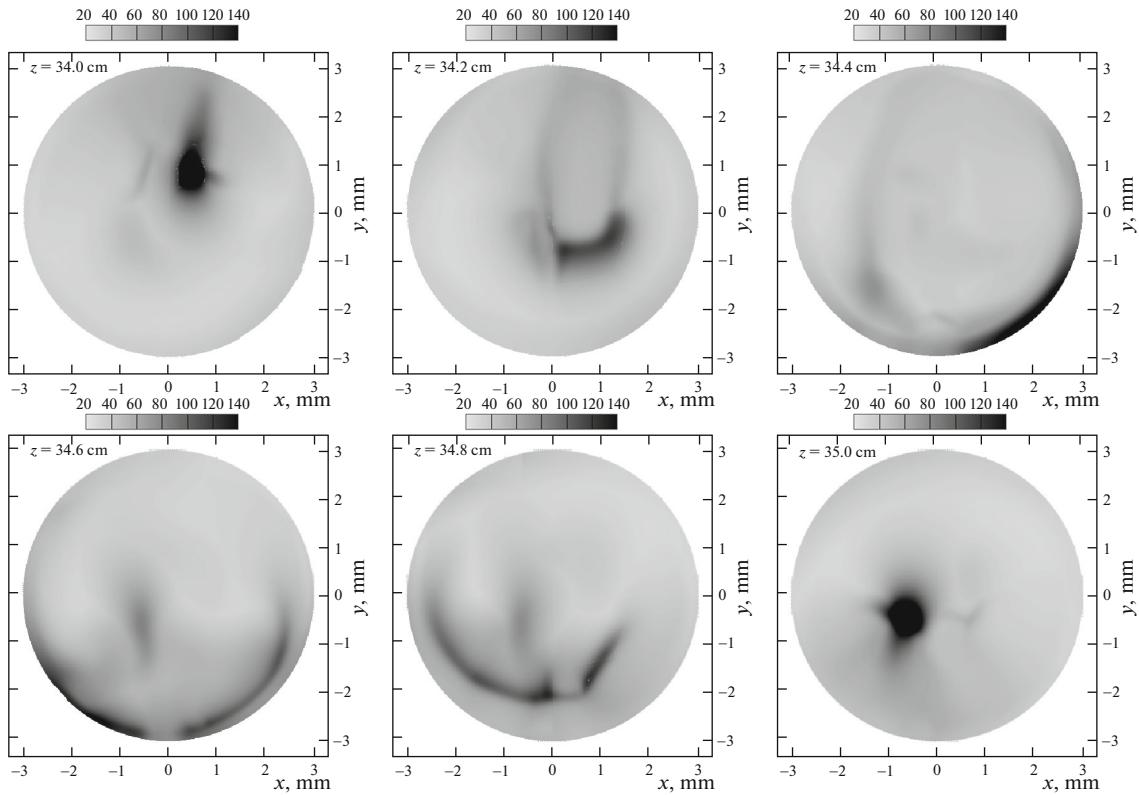
The computations show that the transformation of the one-dimensional detonation into the spinning detonation passes through four phases: (1) propagation of a one-dimensional detonation wave with increasing perturbations occurring due to the limited precision of arithmetic operations; (2) detonation with a chaotic three-dimensional structure caused by intensive transverse waves; (3) gradual transformation of transverse waves into a rotating transverse detonation wave; (4) spinning detonation under which its front uniformly rotates about the symmetry axis and moves along it.

In a typical computation of detonation in the channel with  $D = 6$  mm, the irregular three-dimensional cellular detonation was observed approximately in the range  $23 \text{ cm} < z < 37$  cm. It is convenient to track the propagation of transverse waves by plane transverse cutoffs of the volume numerical trace pattern. Six such cutoffs between  $z = 34$  cm and  $z = 35$  cm with the step 0.2 cm are shown in Fig. 2. It is seen that one or two transverse waves propagate in the section plane while periodically reflecting from the walls and undergoing cumulation at different points of the section. No strict repetition and structure are observed in the transverse wave pattern. Gradually, through numerous oscillations, the transverse waves start to rotate as a whole, and then turn into one rotating spinning detonation wave. Figure 3 illustrates the developed spinning detonation phase under which the maximum pressure fields only rotate from cross section to cross section. Note that the pressure maximum is attained inside the channel rather than on its surface. This is due to the flow structure in the case of spinning detonation (see Fig. 4).

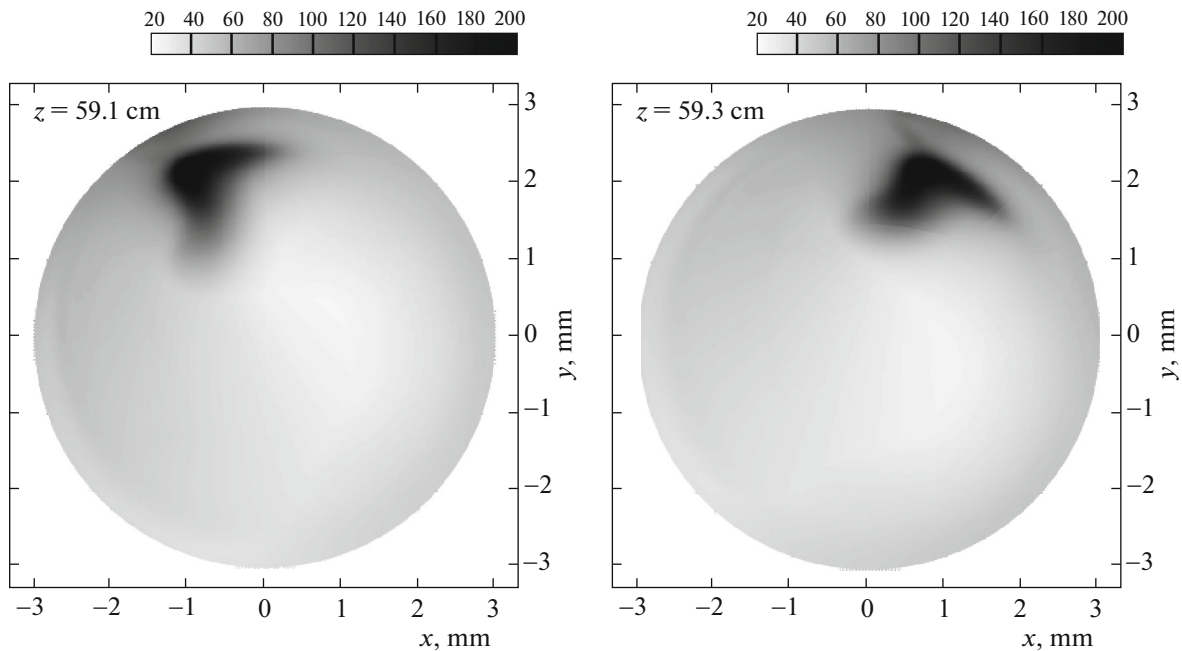
The computations showed that the spinning detonation is characterized by the presence of two main waves—the leading and the transverse ones, which have a common line  $AB$ . Both endpoints of this line move on a spiral. The endpoint  $A$  is on the channel surface, while the other endpoint  $B$  moves inside the channel. The leading front has a kink on the line  $AB$ , and the intensity of cumulation near the point  $B$  is even greater than at the point  $A$  due to a special shape of the leading front and complex interaction between the waves. At the point  $A$ , there are three almost plane interacting waves, while at the point  $B$  a plane wave interacts with an almost conical wave. As a result, the pressure near the leading front inside the channel behind the point  $B$  is greater than behind the point  $A$ .

Note that both the irregular cellular detonation and the spinning detonation have a certain “stability margin,” which lets them exist under small disturbances. Note that spinning detonation can occur not only spontaneously but also as a result of a special initial distribution of gas dynamics parameters in the vicinity of the initiation zone that contains, e.g., a spiral higher temperature region. From this viewpoint, the boundaries of the spinning detonation range presented above are approximate, and they can be interpreted as corresponding to the spontaneous formation of spinning detonation from the one-dimensional detonation under small disturbances.

Note that the solution of two problems with the same initial distribution of parameters but slightly different (at the machine precision level) coordinates of grid nodes or different (at the machine precision level) sequences of time steps results in different small perturbations and, as a consequence, in slightly different flow patterns. However, our computations showed that the sequence of transition of the detonation from one regime to the other one is generally the same, and the transitions occur approximately at the

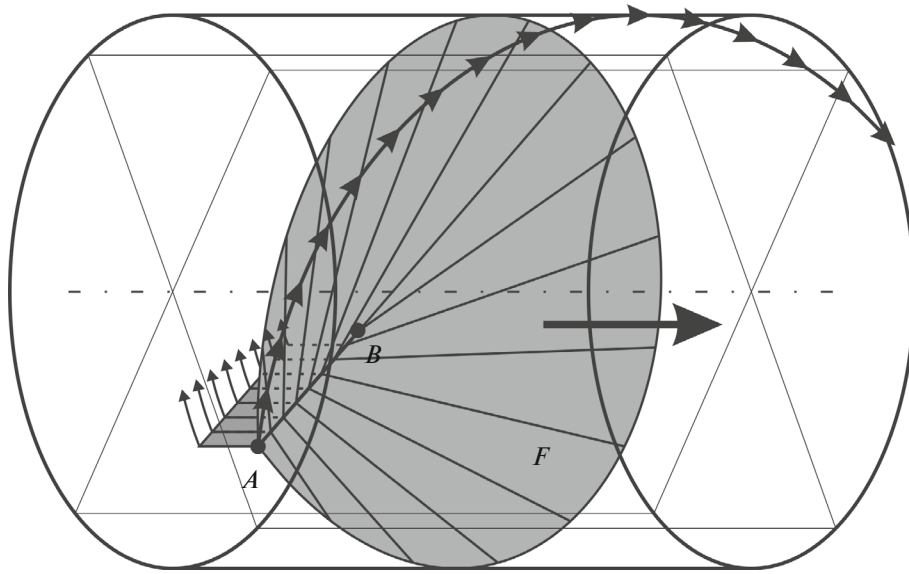


**Fig. 2.** Cutoffs of the numerical trace pattern for  $z = 34, 34.2, 34.4, 34.6, 34.8,$  and  $35$  cm corresponding to the irregular detonation phase preceding the spinning detonation phase.



**Fig. 3.** Cutoffs of the numerical trace pattern for  $z = 59.1$  and  $59.2$  cm corresponding to the developed spinning detonation.

same time. As could be expected from the problem statement, the direction of the spinning detonation rotation is completely undefined and depends on “quasi-random” perturbations and differences of quantities within the machine precision.



**Fig. 4.** The schematic of the spinning detonation propagation:  $F$  is the leading front,  $AB$  is the line of interaction between the leading front and the transverse wave, the point  $A$  is on the channel surface and describes a spiral.

## 5. RESULTS OF THE COMPUTATIONS OF THE SPINNING DETONATION TRANSITION TO THE CHANNEL OF A GREATER OR SMALLER SECTION

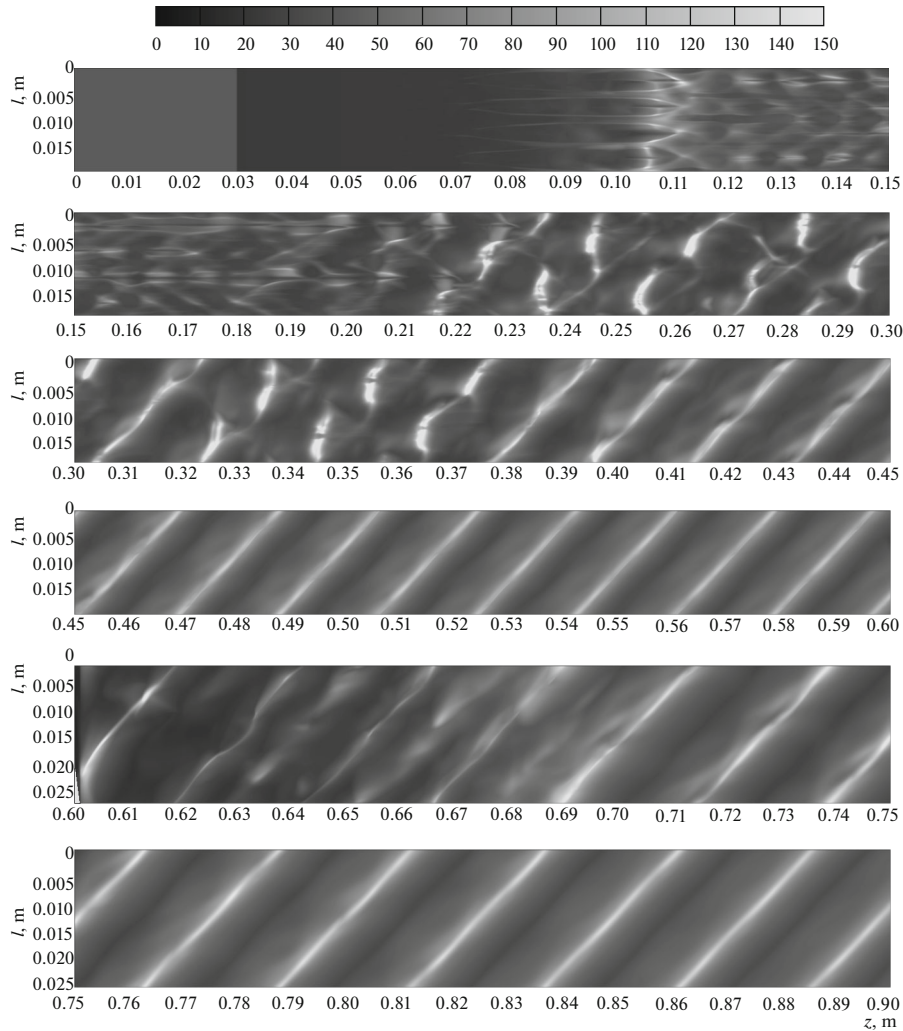
The numerically calculated process of the spontaneous formation of the developed spinning detonation posed the question of its stability under various disturbances. In this section, we discuss the results of the numerical simulation of flows for the case when the spinning detonation passes from a 6 mm channel to a channel with a greater or smaller diameter  $D$  through a short conical segment. A complete description of the geometry of such channels was given above. The length of the 6 mm channel was set as  $L = 60$  cm, which guaranteed the formation of the developed spinning detonation before the conical section.  $D$  is a parameter that significantly affects the flow pattern formed after the wave passes through the conical channel section.

The computations were performed beginning from the time corresponding to the initiation of one-dimensional detonation. This enabled us to obtain and compare the patterns of the formation of spinning detonation in the same channel of 60 cm long and 6 mm in diameter under small various perturbations with the amplitude at the machine precision level. The differences in the perturbations were mainly due to the different sequence of time steps caused by differences in the computational grids.

To analyze the numerically simulated flow patterns, we visualized the developments of the channel lateral surface with numerical trace patterns marked on them. They clearly show the traces of the transverse waves left on the channel surface. The developed spinning detonation can be identified by the characteristic periodically repeating slanted strips.

The typical patterns of the detonation transformation obtained in the computations are shown in Figs. 5–12. For the convenience of perception, the developments of the lateral surface are divided into nonoverlapping fragments of 15 cm long. These fragments are arranged in ascending order of  $z$  from top to bottom.

In the case  $D > 6$  mm, the computations showed the existence of a critical value of the channel diameter  $D_{**} = 8.5$  mm such that the spinning detonation is restored when  $6 \text{ mm} < D < D_{**}$  and disappears for  $D > D_{**}$ . Figure 5 illustrates the process of the detonation propagation for the case  $D = 8$  mm. It is seen that, for  $3 \text{ cm} < z < 22 \text{ cm}$ , small perturbations that result in the irregular three-dimensional cellular detonation regime observed in the range  $22 \text{ cm} < z < 37 \text{ cm}$  are increasing. Then, the transition to the spinning detonation occurs, which becomes developed when it approaches the section  $z = 60$  cm. After the channel becomes wider due to the short conical part in the range  $60 \text{ cm} < z < 60.1 \text{ cm}$ , the spinning detonation partially loses its structure. The weakening of detonation can be observed by lower values of the mean maximum of the pressure (a darker color in the lateral surface development in Fig. 5). In addition, signs of double-headed spin, in which two transverse waves propagate in the same direction, are observed



**Fig. 5.** Development of the lateral surface with the numerical trace pattern obtained for the channel composed of the cylinders with the diameters 6 mm and 8 mm.

in the range  $63 \text{ cm} < z < 69 \text{ cm}$ . This is also confirmed by the left panel of Fig. 6, where the numerical cutoff of the trace pattern for  $z = 66.5 \text{ cm}$  with two dark spots is shown. However, the spinning detonation is completely restored on the last segment  $z > 69 \text{ cm}$  as is seen from the corresponding parts of the lateral surface development. The right panel of Fig. 6 shows the transverse cutoff of the numerical trace pattern for  $z = 88.4 \text{ cm}$ , which is typical for the developed spinning detonation.

For  $D = 10 \text{ mm}$ , there is no spinning detonation after the channel becomes wider. This is illustrated in Fig. 7, where no trace pattern is observed beyond  $z = 63 \text{ cm}$  and the maximum pressure becomes lower. Figure 8 illustrates the structure of the transverse waves almost immediately after the spinning detonation leaves the channel with the smaller diameter; it is also seen that there are no transverse waves at large distances from the point, where the channel becomes narrow.

For the channels with  $D < 6 \text{ mm}$ , the computations also showed the existence of a critical value of the channel diameter  $D_* = 4 \text{ mm}$  such that the spinning detonation is restored after the narrowing when  $D_* < D < 6 \text{ mm}$  and the spin disappears when  $D < D_*$ . Figure 9 illustrates the process of detonation propagation for the case when the spin is restored at  $D = 5 \text{ mm}$ . It is also seen from this figure that the formation of the spinning detonation proceeds similarly to the cases described above. However, the effect of the random perturbations at the machine precision level in this case is that the direction of the spinning detonation rotation is reversed. For the statement of the problem considered in this paper, the direction of the



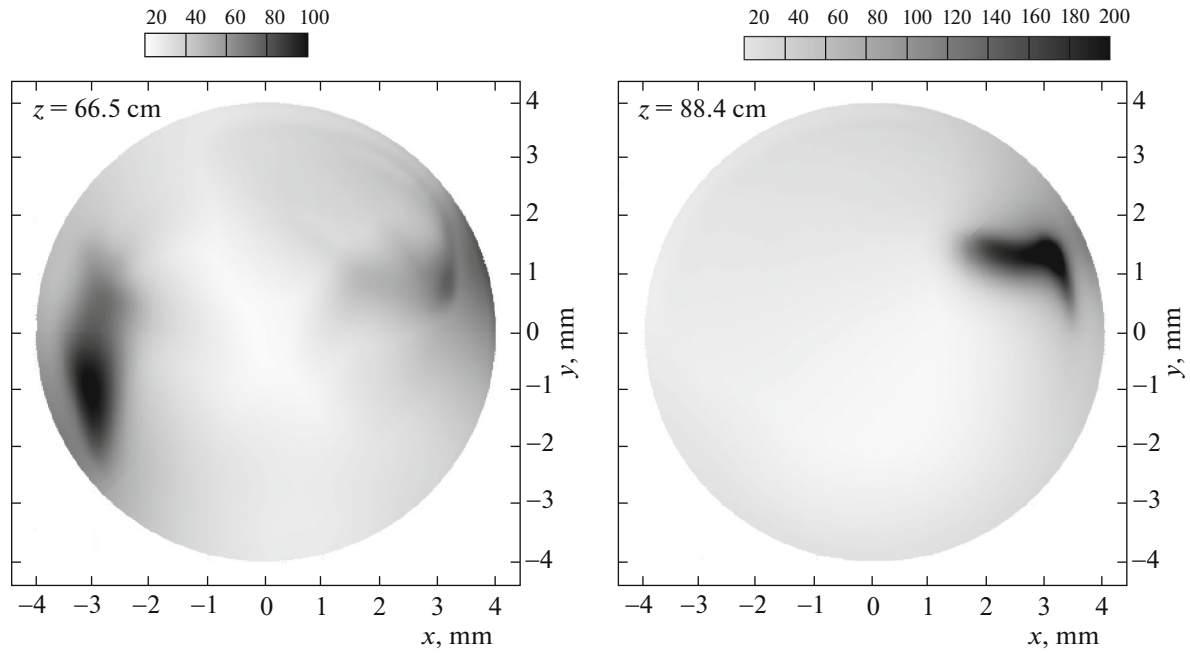


Fig. 6. Cutoffs of the numerical trace pattern for  $z = 66.5 \text{ cm}$  and  $88.4 \text{ cm}$ .

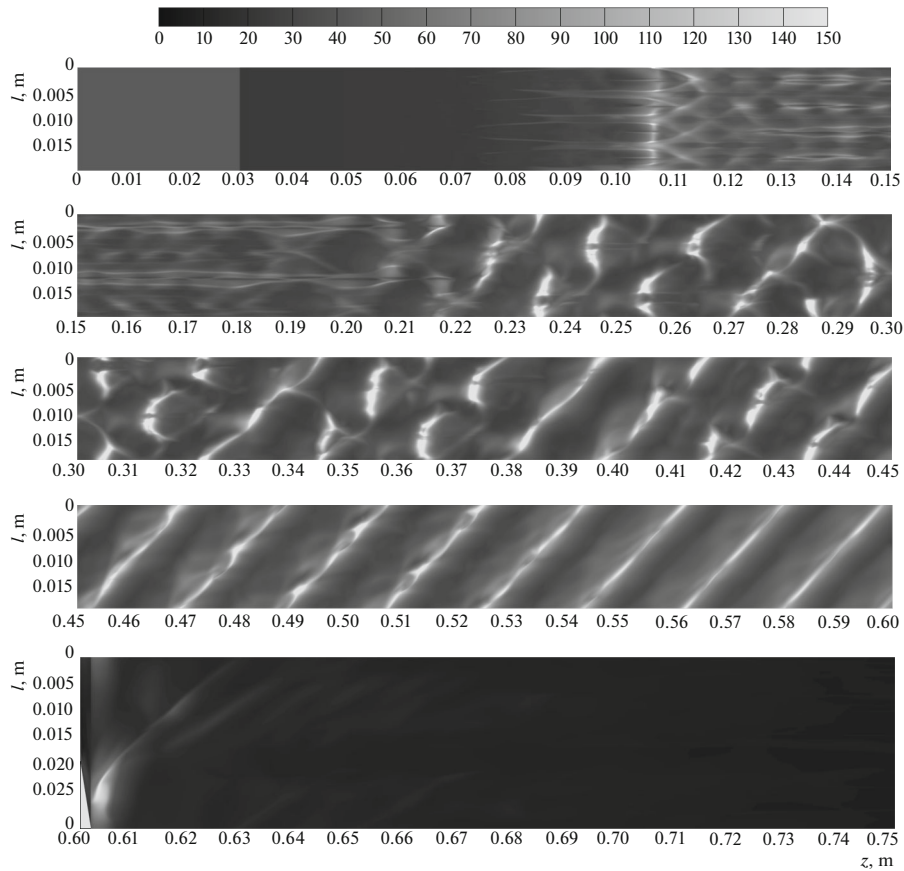
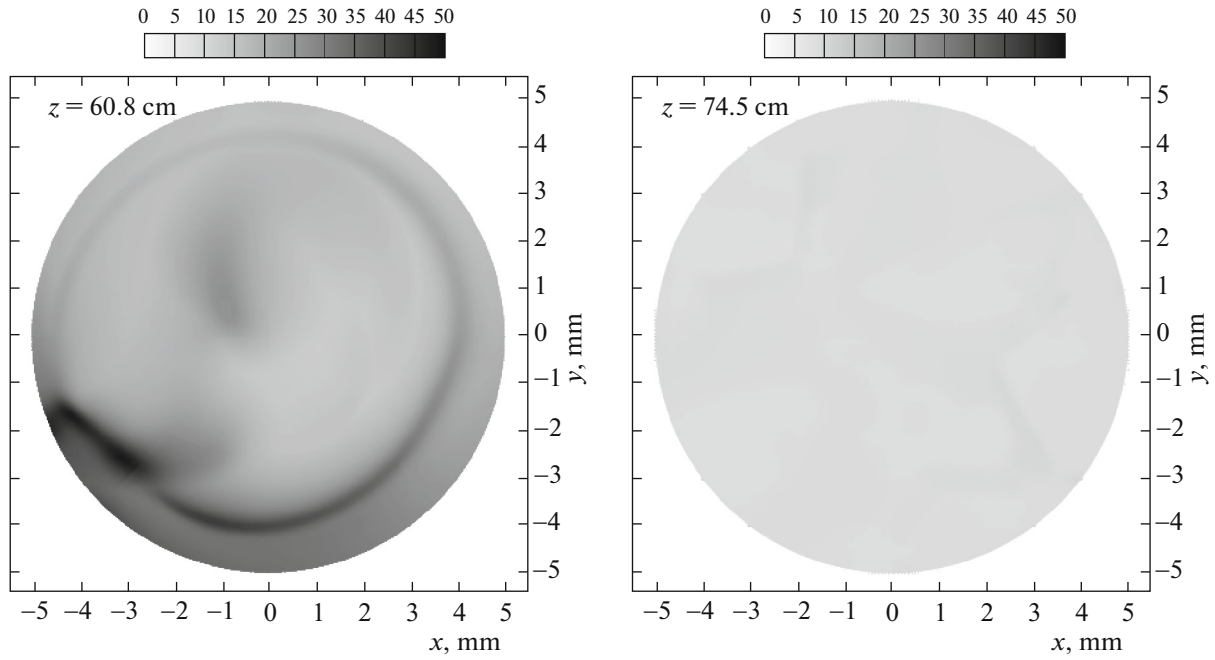
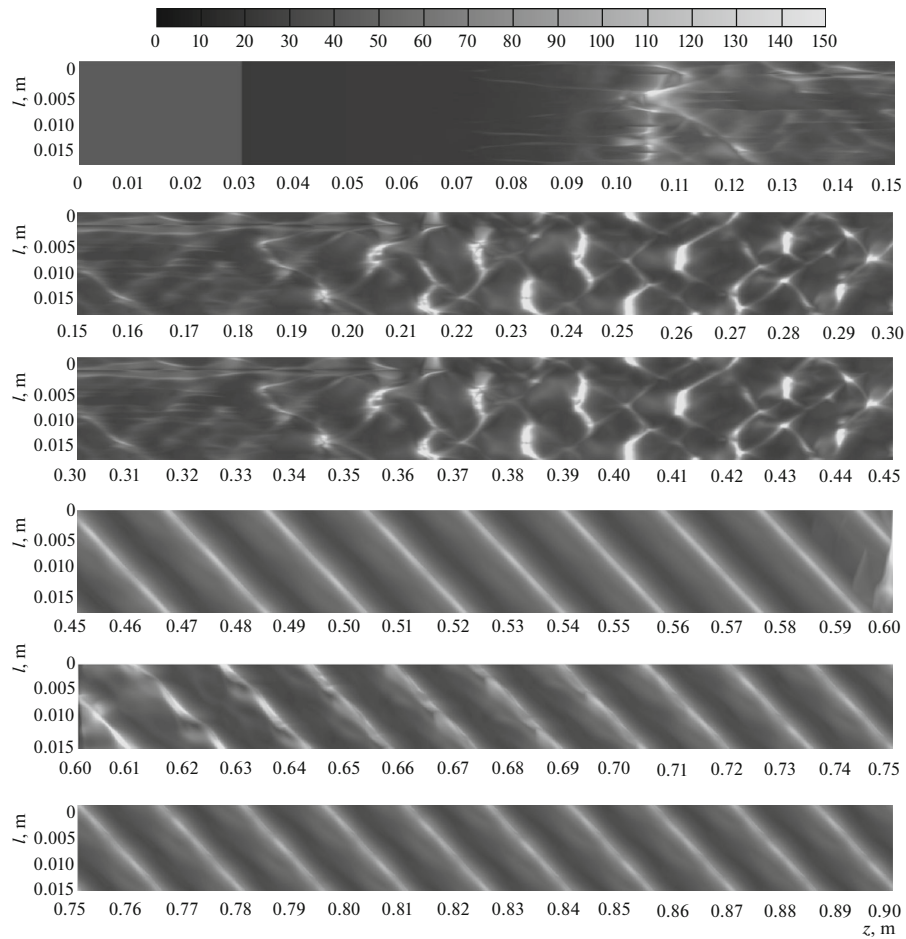


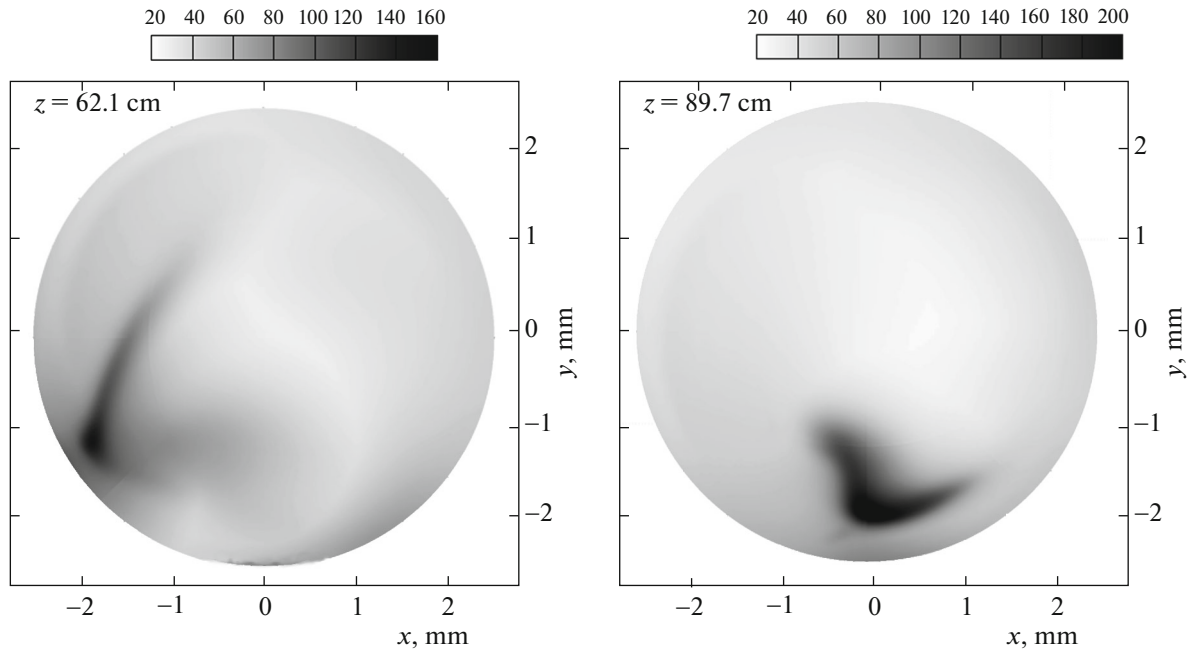
Fig. 7. Development of the lateral surface with the numerical trace pattern obtained for the channel composed of the cylinders with the diameters 6 mm and 10 mm.



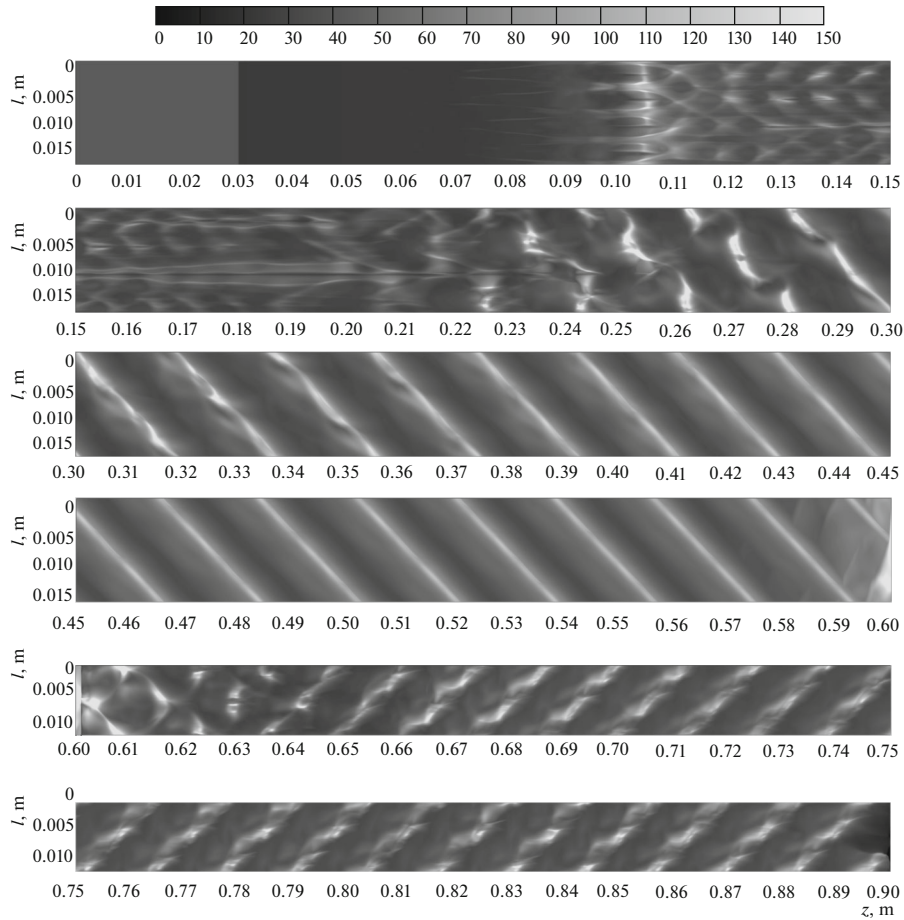
**Fig. 8.** Cutoffs of the numerical trace pattern for  $z = 60.8 \text{ cm}$  and  $74.5 \text{ cm}$ .



**Fig. 9.** Development of the lateral surface with the numerical trace pattern obtained for the channel composed of the cylinders with the diameters 6 mm and 5 mm.



**Fig. 10.** Cutoffs of the numerical trace pattern for  $z = 62.1$  cm and  $89.7$  cm.



**Fig. 11.** Development of the lateral surface with the numerical trace pattern obtained for the channel composed of the cylinders with the diameters 6 mm and 4 mm.

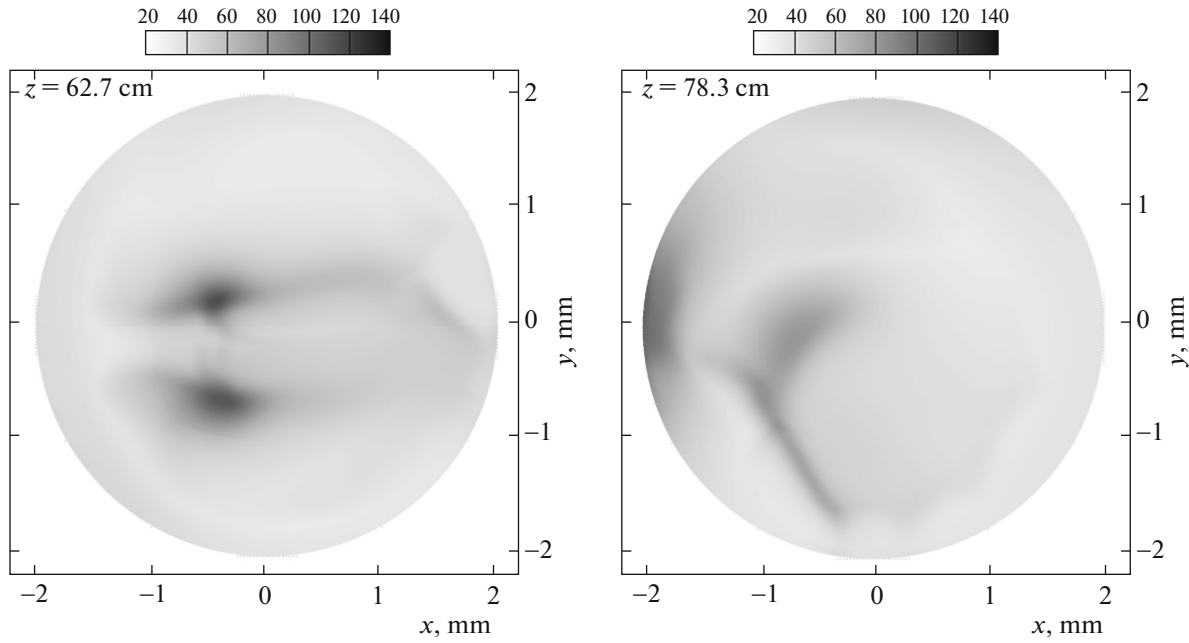


Fig. 12. Cutoffs of the numerical trace pattern for  $z = 62.7$  cm and  $78.3$  cm.

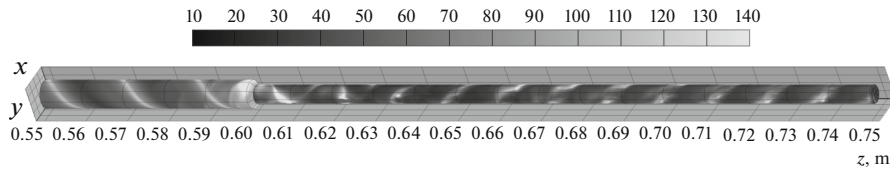


Fig. 13. A fragment of the numerical trace pattern in the case of spinning detonation passing from the channel of the diameter of 6 mm to the 4 mm channel.

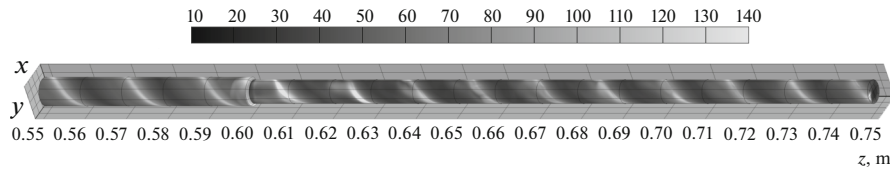
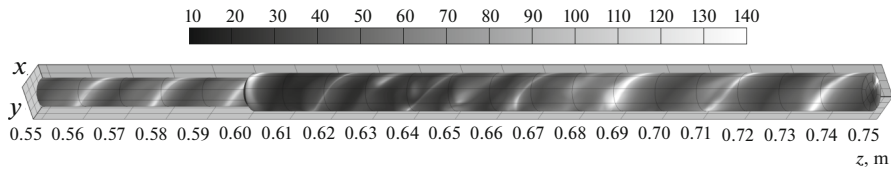


Fig. 14. A fragment of the numerical trace pattern in the case of spinning detonation passing from the channel of the diameter of 6 mm to the 5 mm channel.

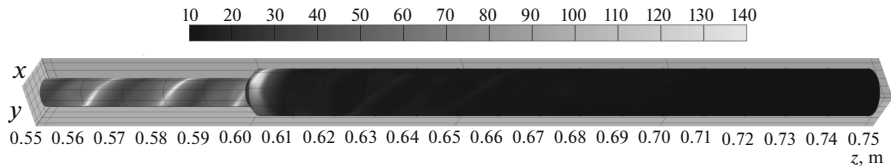
spinning detonation rotation can be arbitrary because the distribution of the initial parameters is symmetric and the small perturbations are not specified and are practically random.

The computations for  $D = 5$  mm show that the spinning detonation is somewhat transformed, which is seen from the irregular slanted line in the range  $60 \text{ cm} < z < 70 \text{ cm}$  in Fig. 9 and by the transverse cutoff of the numerical trace pattern in the left panel of Fig. 10. Farther along the channel, as in the case of the small channel expansion, the disturbances gradually decay and the transition to the developed spinning detonation occurs (the right panel of Fig. 10).

The value  $D = 4$  mm is a boundary one for the regimes with the suppression and restoration of spinning detonation. The numerical results for this value of  $D$  are illustrated in Figs. 11 and 12. In this case, the same direction of rotation of the spinning detonation as in Fig. 9 was observed (the seeming correspondence between the change in the channel diameter and the direction of rotation is a coincidence). Already in the region  $60 \text{ cm} < z < 65 \text{ cm}$ , the mode with two transverse waves without apparent rotation (the left panel



**Fig. 15.** A fragment of the numerical trace pattern in the case of spinning detonation passing from the channel of the diameter of 6 mm to the 8 mm channel.



**Fig. 16.** A fragment of the numerical trace pattern in the case of spinning detonation passing from the channel of the diameter of 6 mm to the 10 mm channel.

in Fig. 12) is observed. Then, the structure of the flow is transformed such that the rotation opposite to the initial one prevails, which is seen from the pattern of motion of the transverse waves and from the approximate slope of light regions in Fig. 11. However, by and large this rotational regime of detonation cannot be called the spinning detonation because it is similar to the irregular cellular detonation regime.

The computations of the developed spinning detonation performed for channels of various diameters show that the period of rotation of the leading front is proportional to the channel diameter. This follows from the identical slope of the lines characterizing the spinning detonation in Figs. 5, 7, 9, and 11 for all channels. It is of interest that this feature is also known from the experiments with the spinning detonation.

We also note once more that, due to different small perturbations at the machine precision level, there are differences in the field of gas dynamic parameters as the wave propagates up to the section  $z = 60$  cm. To verify this fact, one can compare the first four fragments shown in Figs. 5, 7, 9, and 11. However, by and large the initial phases of the detonation propagation illustrated in these figures are identical.

For comparison, Figs. 13–16 show in a three-dimensional form fragments of numerical trace patterns for the spinning detonation that passes from a 6 mm channel to the channels with the diameters of 4, 5, 8, and 10 mm.

## 6. CONCLUSIONS

Using the Lomonosov supercomputer, the three-dimensional gas detonation structures in circular section channels was numerically investigated. Computations show that the structures are formed spontaneously due to the instability when initiated by plane one-dimensional shock wave caused by energy supply at the closed channel end. In channels with a sufficiently large diameter, irregular three-dimensional cellular structure is obtained. It is found that, in circular section channels of a sufficiently small diameter, the initially plane detonation wave spontaneously transforms into a spinning detonation wave while passing through four phases: (1) propagation of a one-dimensional detonation wave with increasing perturbations occurring due to the limited precision of arithmetic operations; (2) detonation with a chaotic three-dimensional structure caused by intensive transverse waves; (3) gradual transformation of transverse waves into a rotating transverse detonation wave; (4) spinning detonation under which its front uniformly rotates about the symmetry axis and moves along it. The critical values of the channel diameter separating the regimes with the three-dimensional cellular spinning detonation, and detonation close to the one-dimensional one are found. It is shown that the direction of spin rotation in the statement considered in this paper is determined by random factors and the rotation period of the leading front is always proportional to the channel diameter.

The stability of the spinning detonation wave under disturbances due to its transition to the channel of a greater (smaller) diameter is investigated. It is found that the spin is preserved if the channel diameter is smaller (or, respectively, greater) than a certain critical value  $D_{**}$  ( $D_*$ ).

All the computations of the cellular and spinning detonation were performed for a long three-dimensional channel (up to 1 m long) as a whole rather than only in its part containing the detonation wave; this made it possible to adequately simulate and investigate the features of the transformation of the detonation structure in the process of its propagation.

The computations were performed using grids with 0.1–10 billions of computational cells. In order to analyze the flow pattern, a numerical trace pattern was constructed based on the pressure maximums, and its cross sections and the development of the channel lateral surface with trace markings, which illustrate the behavior of the transverse waves, was visualized.

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#### REFERENCES

1. R. I. Soloukhin, "On the detonation in the gas heated by a shock wave," *Prikl. Mekh. Tekh. Fiz.*, No. 4, 42–48 (1964).
2. R. I. Soloukhin, "The zone of exothermic reaction in a one-dimensional shock wave in gases," *Fiz. Goreniya Vzryva*, No. 3, 2–18 (1966).
3. R. I. Soloukhin, *Measurement Methods and Basic Results in Experiments on Shock Tubes* (Nauka, Novosibirsk, 1969) [in Russian].
4. R. I. Soloukhin, *Shock Waves and Detonation in Gases* (Fizmatgiz, Moscow, 1963) [in Russian].
5. V. V. Mitrofanov and R. I. Soloukhin, "On the diffraction of multifront detonation wave," *Dokl. Akad. Nauk SSSR* **159**, 1003–1006 (1964).
6. R. I. Soloukhin, "The structure of the multifront detonation wave in gases," *Fiz. Goreniya Vzryva*, No. 2, 35–42 (1965).
7. V. P. Korobeinikov and V. A. Levin, "A strong explosion in a combustible gas mixture," *Izv. Akad. Nauk SSSR, Ser. Mekh. Zhidkosti Gaza*, No. 6, 48–51 (1969).
8. G. G. Chernyi, "An asymptotic law of the plane detonation wave propagation," *Dokl. Akad. Nauk SSSR* **172**, 558–560 (1967).
9. V. A. Levin and G. G. Chernyi, "Asymptotic laws of detonation wave behavior," *Prikl. Mat. Mekh.* **31**, 383–405 (1967).
10. V. P. Korobeinikov, V. A. Levin, V. V. Markov, and G. G. Chernyi, "Propagation of blast waves in a combustible gas," *Astronautica Acta* **17** (5–6), 529–537 (1972).
11. V. A. Levin and V. V. Markov, "On the occurrence of detonation under the concentrated supply of energy," *Izv. Akad. Nauk SSSR, Ser. Mekh. Zhidkosti Gaza*, No. 5, 89–93 (1974).
12. V. A. Levin and V. V. Markov, "Investigation of the occurrence of detonation under the concentrated supply of energy," *Fiz. Goreniya Vzryva* **2**, 623–629 (1975).
13. V. P. Korobeinikov and V. V. Markov, "On propagation of combustion and detonation // *Archiwum procesow spalania* **8** (1), 101–118 (1977).
14. L. I. Sedov, V. P. Korobeinikov, and V. V. Markov, "The theory of blast wave propagation," *Tr. Mat. Inst. im. V.A. Steklova, Akad. Nauk SSSR* **225**, 178–216 (1986).
15. V. A. Levin, V. V. Markov, and S. F. Osinkin, "Direct initiation of detonation in a hydrogen–oxygen mixture diluted with nitrogen," *Izv. Akad. Nauk SSSR, Ser. Mekh. Zhidkosti Gaza*, No. 6, 151–156 (1992).
16. V. A. Levin, V. V. Markov, and S. F. Osinkin, "Simulation of initiation of detonation in inflammable gas mixture by electric charge," *Khim. Fiz.* **3**, 611–613 (1984).
17. V. A. Levin, V. V. Markov, and S. F. Osinkin, "Initiation of detonation in hydrogen–air mixture by the explosion of a spherical TNT charge," *Fiz. Goreniya Vzryva* **31** (2), 91–95 (1995).
18. V. A. Levin, V. V. Markov, and S. F. Osinkin, "Initiation of detonation in the inhomogeneous hydrogen–air mixture," Report No. 4376, Inst. Mekh. RAN (Mosk. Gos. Univ., Moscow, 1995).
19. V. A. Levin, V. V. Markov, and S. F. Osinkin, "Initiation of detonation in the hydrogen–air mixture by an explosive charge surrounded by an inert gas layer," *Vestn. Mosk. Univ., Ser. 1: Mat., Mekh.*, No. 4, 32–34 (1997).
20. V. A. Levin, V. V. Markov, and S. F. Osinkin, "The effect of air interlayer on the Shock Initiation of detonation in a hydrogen–air mixture," *Proc. Steklov Inst. Math.* **223**, 131–138 (1998).

21. V. A. Levin, V. V. Markov, and S. F. Osinkin, "Detonation wave reinitiation using a disintegrating shell," *Dokl. Akad. Nauk SSSR* **352** (1), 48–50 (1997).
22. V. A. Levin, V. V. Markov, and S. F. Osinkin, "The Influence of the disintegrating shell on the initiation of detonation in the hydrogen–air mixture," *XI Symp. on Combustion and Explosion, Chernogolovka, Russia, 1998*, Vol. 2, pp. 169–170.
23. V. A. Levin, V. V. Markov, and S. F. Osinkin, "Stabilization of detonation in supersonic flows of combustible gas mixtures," *Proc. of the 16th Int. Colloquium on the dynamics of explosions and reactive systems, Cracow, Poland, 1997*, pp. 529–537.
24. V. A. Levin, V. V. Markov, and T. A. Zhuravskaya, "Direct initiation of detonation in hydrogen air mixtures by decomposition of low pressure domain without energy input," *Archivum combustionis* **18** (1-4), 125–133 (1998).
25. V. A. Levin, V. V. Markov, and T. A. Zhuravskaya, "Direct initiation of detonation in a hydrogen–air mixture by a converging shock wave," *Khim. Fiz.* **20** (5), 26–30 (2001).
26. V. A. Levin, V. V. Markov, S. F. Osinkin, and T. A. Zhuravskaya, "Determination of critical conditions of initiation of detonation in a bounded volume by shock wave converging to the center," *Fiz. Goreniya Vzryva* **38** (6), 96–102 (2002).
27. T. A. Zhuravskaya, V. A. Levin, V. V. Markov, and S. F. Osinkin, "Influence of the destructible shell on the formation of detonation in a bounded volume by a converging shock wave," *Khim. Fiz.* **22** (8), 34–37 (2003).
28. V. V. Markov, "Numerical simulation of the formation of multifront structure of the detonation wave," *Dokl. Akad. Nauk SSSR* **258** (2), 158–163 (1981).
29. V. A. Levin, V. V. Markov, T. A. Zhuravskaya, and S. F. Osinkin, "Nonlinear wave processes that occur during the initiation and propagation of gaseous detonation," *Proc. Steklov Inst. Math.* **251**, 192–205 (2005).
30. V. A. Levin, V. V. Markov, T. A. Zhuravskaya, and S. F. Osinkin, "Initiation of gas detonation by electric discharges," in *Pulse Detonation Engines* (TORUS, Moscow, 2006), pp. 120–138.
31. V. A. Levin, V. V. Markov, T. A. Zhuravskaya, and S. F. Osinkin, "Initiation and propagation of detonation in channels of complex shape," in *Pulse and Continuous Detonation Propulsion*, Ed. by G. D. Roy and S. M. Frolov Frolov (TORUS, Moscow, 2006), pp. 97–106.
32. V. A. Levin, V. V. Markov, T. A. Zhuravskaya, and S. F. Osinkin, "Determination of the critical conditions of the propagation of detonation waves in channels of complex shapes," in *Modern Problems of Fast Processes and Catastrophic Events*, Ed. by O. M. Belotserkovskii (Nauka, Moscow, 2007), pp. 75–88 [in Russian].
33. Levin V.A., Markov V.V., T. A. Zhuravskaya, and S. F. Osinkin, "Influence of obstacles on detonation wave propagation," in *Deflagrative and Detonative Combustion*, Ed. by G. Roy and S. Frolov (TORUS, Moscow, 2010), pp. 221–228.
34. V. A. Levin, V. V. Markov, T. A. Zhuravskaya, and S. F. Osinkin, "Initiation, propagation, and stabilization of detonation waves in a supersonic stream," in *Problems of Modern Mechanics* (Mosk. Gos. Univ., Moscow, 2008), pp. 240–259 [in Russian].
35. V. A. Levin, V. V. Markov, T. A. Zhuravskaya, and S. F. Osinkin, "Initiation, propagation and stabilization of detonation in the supersonic gas flow," *Proc. of the 7th Int. Symposium on Hazards Prevention and Migration of Industrial Explosions (ISHPMIE), St. Petersburg, Russia, 2008*, Vol. 2, pp. 110–118.
36. V. A. Levin, V. V. Markov, and A. N. Khmelevskii, "Theoretical and experimental investigation of the operation of a pulse detonation engine," *Khim. Fiz.* **24** (7), 37–43 (2005).
37. E. M. Barkhudarov and N. K. Berezhetskaya, T. A. Zhuravskaya, V. A. Kop'ev, I. A. Kosygi, V. A. Levin, V. V. Markov, N. A. Popov, M. I. Taktakishvili, N. M. Tarasova, and S. M. Temchin, "An axisymmetric electric discharge as a means for remote heating of gas and for ignition of combustible gas mixture," *Fizika Plazmy* **35**, 1001–1010 (2009).
38. V. A. Levin, I. S. Manuylovich, and V. V. Markov, "Optimization of the thrust performance of a pulsed detonation engine," *Combust., Explos., Shock Waves* **46**, 418–425 (2010).
39. V. A. Levin, I. S. Manuylovich, and V. V. Markov, "New effects of stratified gas detonation," *Dokl. Phys.* **55**, 28–32 (2010).
40. V. A. Levin, I. S. Manuylovich, and V. V. Markov, "Distinctive features of galloping detonation in a supersonic combustible-mixture flow under an inert gas layer," *Fluid Dyn.* **45**, 827–835 (2010).
41. V. A. Levin, I. S. Manuylovich, and V. V. Markov, "Formation of Detonation in Rotating Channels," *Dokl. Phys.* **55**, 308–311 (2010).
42. V. A. Levin, I. S. Manuylovich, and V. V. Markov, "Detonation initiation by rotation of an elliptic cylinder inside a circular cylinder and deformation of the channel walls," *J. Appl. Mech. Tech. Phys.* **51**, 463–470 (2010).

43. V. A. Levin, N. E. Afonina, V. G. Gromov, G. D. Smekhov, A. N. Khmelevsky, and V. V. Markov, "Gas dynamics and thrust in the exhaust system of a jet engine with an annular nozzle," *Combust., Explos., Shock Waves* **48** (4), 406–417 (2012).
44. V. A. Levin, N. E. Afonina, V. G. Gromov, G. D. Smekhov, A. N. Khmelevsky, and V. V. Markov, "Investigating an annular nozzle on combustion products of hydrocarbon fuels," *Thermophys. Aeromech.* **20**, 265–272 (2013).
45. V. A. Levin, I. S. Manuylovich, and V. V. Markov, "Mathematical modeling of shock-wave processes under gas–solid boundary interaction," *Proc. Steklov Inst. Math.* **281**, 37–48 (2013).
46. V. A. Levin, N. E. Afonina, V. G. Gromov, I. S. Manuylovich, V. V. Markov, G. D. Smekhov, and A. N. Khmelevskii, "Experimental and numerical simulation of the flow in a driving module with an annular and linear double-slot nozzle," *High Temp.* **51**, 681–689 (2013).
47. L. V. Gurvich, G. A. Khachkuruzov, V. A. Medvedev, et al., *Thermodynamic Properties of Individual Substances: Handbook, Vol. 1.* (Nauka, Moscow, 1978) [in Russian].
48. C. K. Westbrook and F. L. Dryer, "Chemical kinetic modeling of hydrocarbon combustion," *Prog. Energy Combust. Sci.* **10**, 1–57 (1984).
49. S. K. Godunov, A. V. Zabrodin, M. Ya. Ivanov, A. N. Kraiko, and G. P. Prokopov, *Numerical Solution of Multidimensional Fluid Dynamics Problems* (Nauka, Moscow, 1976) [in Russian].
50. V. Voevodin, S. Zhumatii, S. Sobolev, A. Antonov, P. Bryzgalov, D. Nikitenko, K. Stefanov, and V. Voevodin, "Practice of the Lomonosov supercomputer," *Otkrytye Sist.*, No. 7, 36–39 (2012).

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